1972

Analysis of initially bent two-span columns.

Ploeger, Robert Bowers.

Monterey, California. Naval Postgraduate School

ANALYSIS OF INITIALLY BENT TWO-SPAN COLUMNS

Robert Bowers Ploeger

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

ANALYSIS OF INITIALLY BENT TWO-SPAN COLUMNS

by

Robert Bowers Ploeger

Thesis Advisor: J. E. Brock

June 1972

Analysis of Initially Bent Two-Span Columns

by

Robert Bowers Ploeger
Lieutenant, United States Navy
B.S., United States Naval Academy, 1966

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1972

ABSTRACT

A study of single-span initially bent columns is made and extended to include two-span columns. The axial force in these columns results from heating (or an equivalent) and from bowing. A snap-through phenomenon is considered in the analysis of the single-span and observations are made concerning snap-through in the two-span column. A digital computer program is developed to carry out the analysis and a description of the program enables its use by anyone familiar with the FORTRAN language.

TABLE OF CONTENTS

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Professor John E. Brock, Mechanical Engineering Department, Naval Postgraduate School, for his invaluable assistance in the development and writing of this thesis.

I. INTRODUCTION

A. GENERAL REMARKS

The subject of initially bent columns and shallow arches is treated extensively in the literature. Hsu [Ref. 6] studied the dynamic stability of shallow arches against snap-through; Hoff and Bruce [Ref. 5] analyzed dynamically the snap-through of shallow arches; Fung and Kaplan [Ref. 2] studied the buckling of low arches or curved beams of small curvature; Timoshenko [Ref. 8] has also discussed the problem; Huddleston [Ref. 7] studied the effect of axial strain in elastic columns; Gjelsvik and Bodner [Ref. 3] approached the problem on the basis of energy criteria. There have been other similar studies made. However, these treatments of the problem have all involved only a single span. Brock [Ref. 1] studied the single span using Fourier analysis in such a way as to provide for assembly into a multispan continuous column, and gave conditions for the solution of multispan cases. However, he did not actually arrive at numerical solution in any particular case, only suggesting how the computations might be carried out. Therefore, it was the objective of this thesis not only to study the single-span column in greater depth, but to develop its analysis in a manner such that it could most easily be extended to a multi-span problem, and actually to make this extension to the case of a two-span column.

B. SCOPE OF WORK

The theory of the single-span problem is first developed for a column which is initially nearly straight. This development takes into consideration an axial strain induced in the column and also allows for the external application of end moments. It is assumed that the configuration, with and without load, lies in a single plane. The theory is then extended to include the two-span column. A digital computer program is developed to carry out the analysis, and a description of the program enables its use by anyone who is familiar with the FORTRAN language.

Analysis beyond the two-span problem is not carried out in this thesis, although it is envisioned that multi-span problems can be investigated in much the same manner as described herein.

C. NOTATION

A - cross sectional area

a - Fourier coefficients for initial configuration

b - Fourier coefficients for final configuration

E - Young's modulus of elasticity

I - moment of inertia of column

L - length of span

M - left end moment

N - right end moment

n - term index in Fourier analysis

P - axial compressive load

P_c - Euler's critical buckling load

Q - left end reaction force

R - right end reaction force

W - lateral load in two-span column

x - distance from left end of span

y_o - initial deflection

y - final deflection

ε - unit strain

ε_T - thermal unit strain

κ - curvature

λ_o - excess of arc length over chord length in initial configuration

λ - excess of arc length over chord length in final configuration

π - 3.141592 ...

θ - end slope of span

II. DEVELOPMENT OF THEORY

A. SINGLE SPAN

The study of the single span involved an initially bent column loaded as shown below:

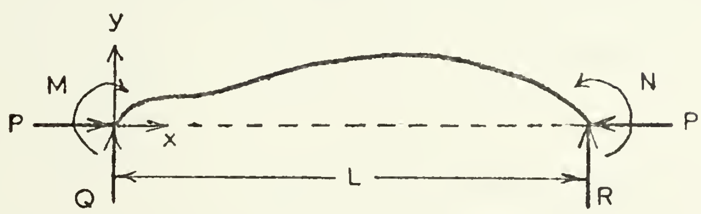


Figure 1. Loading on single span column

The end supports are frictionless hinges which do not move from their original positions. The moment loading shown is applied by external agencies. The initial, no-load configuration of the column is presumed to be given by the equation

$$y_o = \Sigma a_n \sin\left(\frac{n\pi x}{L}\right), \tag{1}$$

where here and later the summation is with respect to n and goes from n=1 to n= ∞. For computational purposes the upper limit is taken to be a finite number, which will be discussed in a later chapter.

Next suppose that the column is uniformly heated so as to provide thermal strain. (It should be noted that an equivalent situation would result from moving the ends of the column toward each other. However, throughout, we will speak in terms of heating the column rather than displacing its ends.) The new configuration can be presumed to be represented by the equation

$$y = \Sigma b_n \sin\left(\frac{n\pi x}{L}\right). \tag{2}$$

The immediate purpose is to find the b_n's and thereby the new configuration.

There is an axial compressive force P set up in the column such that the unit strain is

$$\varepsilon = \varepsilon_T - P/AE \tag{3}$$

where ε_T is the thermal strain, A is the cross section area, and E is Young's modulus of elasticity. (Throughout, we presume that Hooke's law is satisfied.)

The excess of arc length over chord length can be adequately represented for the initial deflection by the approximate relation

$$\lambda_o = \frac{1}{2}\int_0^L \left(\frac{dy}{dx}\right)^2 dx = \frac{\pi^2}{4L} \Sigma\, n^2 a_n{}^2 . \tag{4}$$

Likewise for the new configuration

$$\lambda = \frac{\pi^2}{4L} \Sigma\, n^2 b_n{}^2 . \tag{5}$$

Thus, the change in length due to heating is

$$\lambda - \lambda_o = \left(\frac{\pi^2}{4L}\right) \Sigma\, n^2 (b_n{}^2 - a_n{}^2) = L\varepsilon_T - PL/AE. \tag{6}$$

The bending behavior of the column must also be accounted for. By static equilibrium

$$Q = -R = (N-M)/L. \tag{7}$$

The bending moment at any point is given by

$$B.M. = QX + M - Py, \tag{8}$$

and this accounts for the change in curvature

$$\Delta\kappa = \frac{d^2y}{dx^2} - \frac{d^2y_o}{dx^2} = -\left(\frac{\pi}{L}\right)^2 \Sigma\, n^2 (b_n - a_n) \sin\left(\frac{n\pi x}{L}\right). \tag{9}$$

According to the elementary formula,

$$B.M. = EI\,\Delta\kappa. \tag{10}$$

Thus, with a little manipulation, we find

$$\Sigma\ [n^2 P_c (b_n - a_n) - Pb_n] \sin\left(\frac{n\pi x}{L}\right) + Qx + M = 0, \tag{11}$$

where we have introduced the notation

$$P_c = \pi^2 EI/L^2 . \tag{12}$$

This is the Euler critical load which appears naturally in this calculation. To deal successfully with Eq. (11) it is useful to express Qx and M as Fourier series in the range $0 \leq x \leq L$.

$$Qx = (2QL/\pi)\ \Sigma(-1)^{n+1}\ (1/n) \sin\left(\frac{n\pi x}{L}\right) \tag{13}$$

$$M = (2M/\pi)\ \Sigma[1 - (-i)^n]\ (1/n) \sin\left(\frac{n\pi x}{L}\right). \tag{14}$$

Thus,

$$0 = \Sigma\left\{(-1)^{n+1}\left(\frac{2QL}{n\pi}\right) + [1-(-1)^n]\left(\frac{2M}{n\pi}\right) + n^2 P_c (b_n - a_n) - Pb_n\right\} \sin\left(\frac{n\pi x}{L}\right), \tag{15}$$

which will be satisfied if each of the terms in braces vanishes. Accordingly, we set

$$b_n = \left\{n^2 a_n P_c + (-1)^n\left(\frac{2QL}{n\pi}\right) + [(-1)^n - 1]\left(\frac{2M}{n\pi}\right)\right\}\frac{1}{(n^2 P_c - P)}\ ,\ n = 1,2,\ldots \tag{16}$$

However this equation still involves the unknown, P. Equation (6) can be written in the form

$$P = AE\varepsilon_T - \pi^2 AE/4L^2\ \Sigma\ n^2 (b_n{}^2 - a_n{}^2). \tag{17}$$

Rewriting Eq. (17) once more in the form

$$f(P) = 0, \tag{18}$$

where

$$f(P) = PL/AE + (\pi^2/4L)\Sigma n^2 (b_n^2 - a_n^2) - L\varepsilon_T, \tag{19}$$

we seek the zero (or zeros) of the nonlinear function $f(P)$. Because of nonlinearity, computational difficulties were encountered in finding the significant root. Of several methods attempted, the most suitable was the Newton-Raphson iteration scheme. A study of the curve $f(P)$ revealed that the method would usually converge on the first root if the iteration was started at a value just slightly smaller than P_c.

The value of P thus being known, Eqs. (16) can then be solved for the coefficients b_n.

Figure 2 shows the initial and final deflection curves of a column computed using this analysis. The final deflection shows the column after having undergone thermal strain. The end moments were zero for this simple case.

Further analysis of the single span problem will be given in Section III of the thesis.

B. TWO SPAN

We now consider two such spans, the column being joined at the intermediate support so as to maintain continuity of slope and moment as well as of deflection. In general, different axial loads will be acting in the two spans. As before, the problem is to determine the final configuration after heating, the initial configuration having been specified.

The obvious method of analysis is, in effect, to consider two adjacent single spans, each of which is as shown in Fig. 1 and is analyzed as in the preceeding subsection. The adjoined spans are as shown in Fig. 3. Separating them and considering their mutual

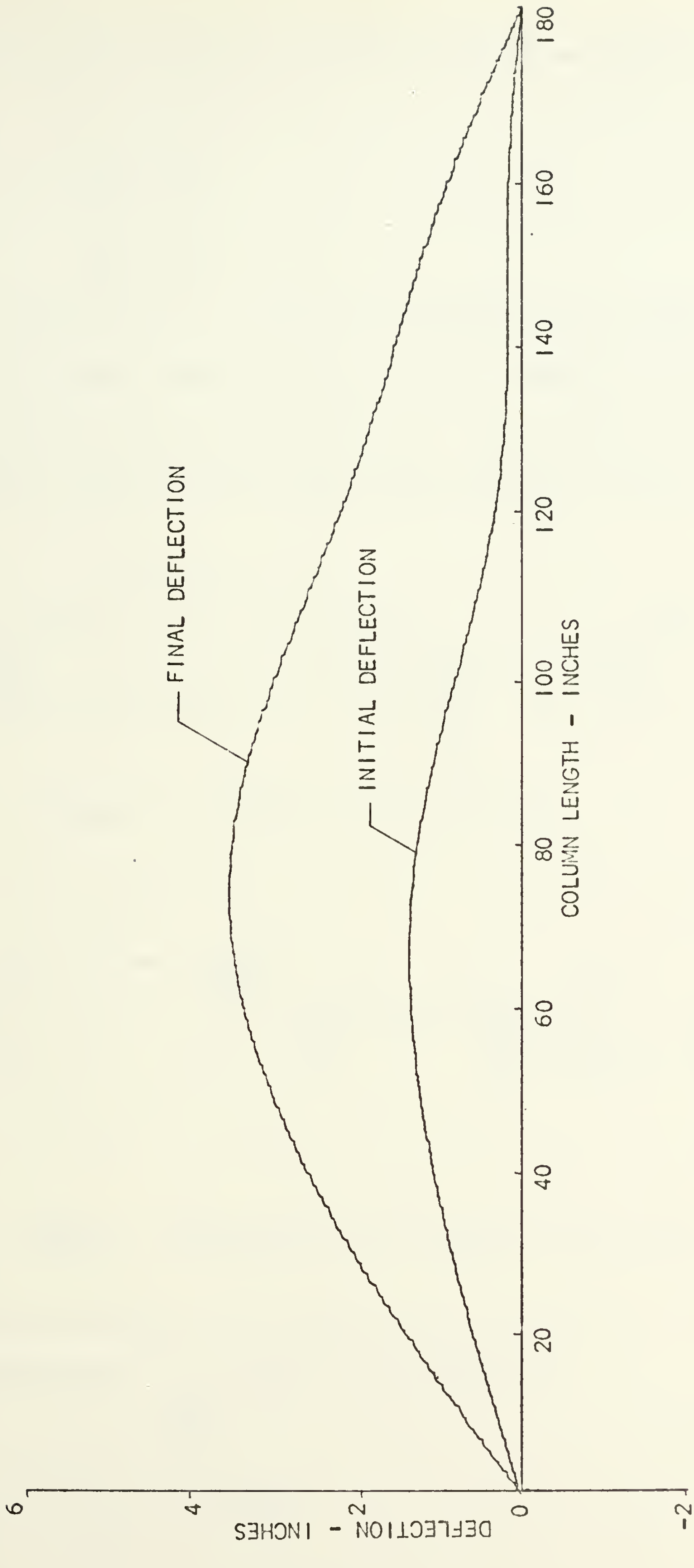


FIGURE 2. SINGLE-SPAN COLUMN UNDERGOING THERMAL STRAIN

interaction and the continuity at their junction, we have (see Fig. 4)

$$N_1 = M_2, \tag{20}$$

$$\theta_{R1} = \theta_{L2} . \tag{21}$$

If the common value $N_1 = M_2$ can be determined, the problem is solved in the sense that each span can be now analyzed separately. As a matter of fact, in the solution for $N_1 = M_2$, we will actually have determined P_1 and P_2 and the Fourier coefficients b_n for each span.

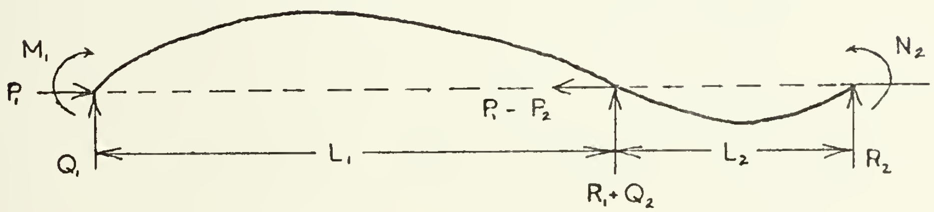


Figure 3. Loading on two-span column

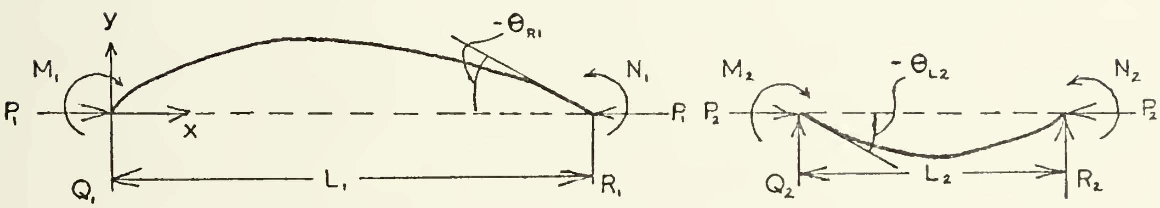


Figure 4. Separation into two single-span columns

The left and right end slopes of each span may be found by evaluating dy/dx at x = 0 and x = L, respectively.

$$\theta_L = \Sigma \frac{n\pi b_n}{L} , \tag{22}$$

$$\theta_R = \Sigma(-1)^n \frac{n\pi b_n}{L} . \tag{23}$$

A method of solution for the two span problem is outlined as follows. First assume a common value for M_1 and M_2. (This will be called the junction moment.) Then analyze each span separately, using the analysis of the preceding section, and determine the slopes at the junction. Compare the two slopes and find the difference between them, which will be called error. Next repeat the above for another value of moment and find the error. A plot of error versus moment will indicate the value of junction moment that will cause the error (in slope) to be zero. At this point, correct values for the Fourier coefficients b_n will be known for each span so that the final configuration can then be computed and plotted if required.

If the plot of slope error versus junction moment is nearly linear, a slope intercept method of iteration may be used to determine where the error goes to zero. This method simply takes two points, gets the equation of the line passing through these points, and then solves for the zero of this linear approximation. This is repeated, using a new point and one of the old points, until the desired convergence is reached. If the plot of slope error versus junction moment is significantly nonlinear (this occurs if the axial compressive load is a large fraction of the Euler load), then the slope intercept method is no longer satisfactory and a method such as successive decimation must be used. In this context, decimation means recalculating over an interval of interest, using increments one tenth as large as previously. The program decimates four times so that the final increment is .0001 times the initial increment. This method is well known and will not be described here. The accuracy of results obtained by using this method depends upon the size of the increment used in stepping. The smaller the increment, the greater the accuracy.

The question here is where to start the iteration. If a good approximate figure is known, then the increments may be small. If not, the increments must be larger. Unfortunately there is no quick way of determining a good starting approximation.

C. INITIAL DEFLECTION

For the single span problem, any initial configuration may be specified subject only to the condition $y(0)=y(L)=0$. If the configuration is specified in the form of a polynomial in x, the axial distance from the left end, the procedure to be described later quickly determines as many of the Fourier coefficients a_n as may be desired.

However, for the two span problem the necessity for continuity places extra requirements on the initial deflection. Not only must the deflection be zero at each end of the spans, but the slope and curvature must be continuous at the junction. Accordingly, such a two span configuration is obtained as follows. An arbitrary configuration (conveniently in the form of a polynomial) is assumed which has zero displacement at the left of span one and at the right of span two. (see Fig. 5). Then, regarding this as a simply supported beam, sufficient lateral load is applied at the intermediate support to cause the displacement to vanish at this point. The corresponding deflection curve is calculated and its superposition with the originally assumed configuration gives a result which now has zero deflection at each of the three support points. In general, the column will thus start out with non-zero bending moment except at the extreme ends.

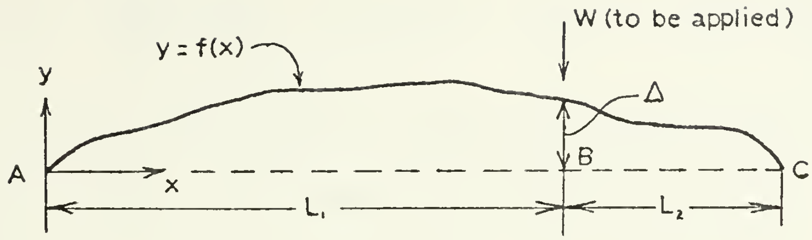


Figure 5. Original specified configuration

Figure 5 illustrates the originally specified configuration before the intermediate constraint is enforced. It is not difficult to show that the necessary lateral load is given by

$$\text{Force} = W = \frac{3(L_1+L_2)\ EI\Delta}{L_1^2 L_2^2} \quad \text{downward.} \tag{24}$$

It can also easily be shown that the deflection of a beam due to a downward force W at point B is represented by

$$-\frac{WL_2}{6EI(L_1+L_2)}\left[(L_1+L_2)^2\,x - x^3 - L_2^2 x + \frac{(L_1+L_2)}{L_2}\ \langle x - L_1\rangle^3\right] \tag{25}$$

where the < > symbol has the meaning

$$\langle x - L_1\rangle^3 = \begin{cases} 0 & \text{if } x \leq L_1 \\ (x-L_1)^3 & \text{if } x \geq L_1 \ . \end{cases}$$

Thus, by superposition, the new configuration is

$$y = f(x) - \frac{WL_2}{6EI(L_1+L_2)}\left[(L_1+L_2)^2\,x - x^3 - L_2^2 x + \frac{(L_1+L_2)}{L_2}\ \langle x - L_1\rangle^3\right]. \tag{26}$$

For the first span the expression for the configuration is

$$y = f(x) - \frac{WL_2\,x}{6EI(L_1+L_2)}\left[(L_1+L_2)^2 - x^2 - L_2^2\right], \tag{27}$$

where $0 \leq x \leq L_1$.

For the second span

$$y = f(x^*+L_1) - \frac{WL_2(x^*+L_1)}{6EI(L_1+L_2)}\left[(L_1+L_2)^2 - (x^*+L_1)^2 - L_2^2 + \frac{(L_1+L_2)}{L_2(x^*+L_1)}x^{*3}\right] \quad (28)$$

where $x^* = x-L_1$ is the distance along span two from its own left end.

Now the Fourier coefficients can be found for each span. If f(x) is a polynomial, the program described in Appendix A makes the determination. Thus we have a deflection satisfying the necessary initial conditions for the two span problem.

III. DISCUSSION

A. ANALYSIS OF THE PROBLEM

In analysis of the single-span problem, the effects of the application of end moments were investigated. This means that end moments were externally applied at one or both ends of the column with or without the presence of axial strain. It was found that a snap-through phenomenon could be observed using this analysis. That is, by increasing one or both end moments, the configuration could be seen to "jump" from one configuration to a distinctly different configuration with only infinitesimal variation in moment. Figure 6 illustrates this phenomenon by showing a column that is first heated up, and then subject to a left end moment. Curve 1 represents the initial configuration of the column. Curve 2 is the configuration of the column after undergoing thermal strain. The remaining curves show the configuration under the action of several different values of end moment. It is seen that the first few 1000 inch pound increments of applied moment cause only slight changes in configuration, but that the change from 6000 to 7000 causes a relatively large change which has the nature of a jump, or snap-through phenomenon, following which addition of increments of applied moment again result in only minor adjustments of the configuration.

Figure 7 further illustrates the phenomenon by showing a plot of centerline deflection versus time, where time is used in the sense that first thermal strain is increased uniformly, with no

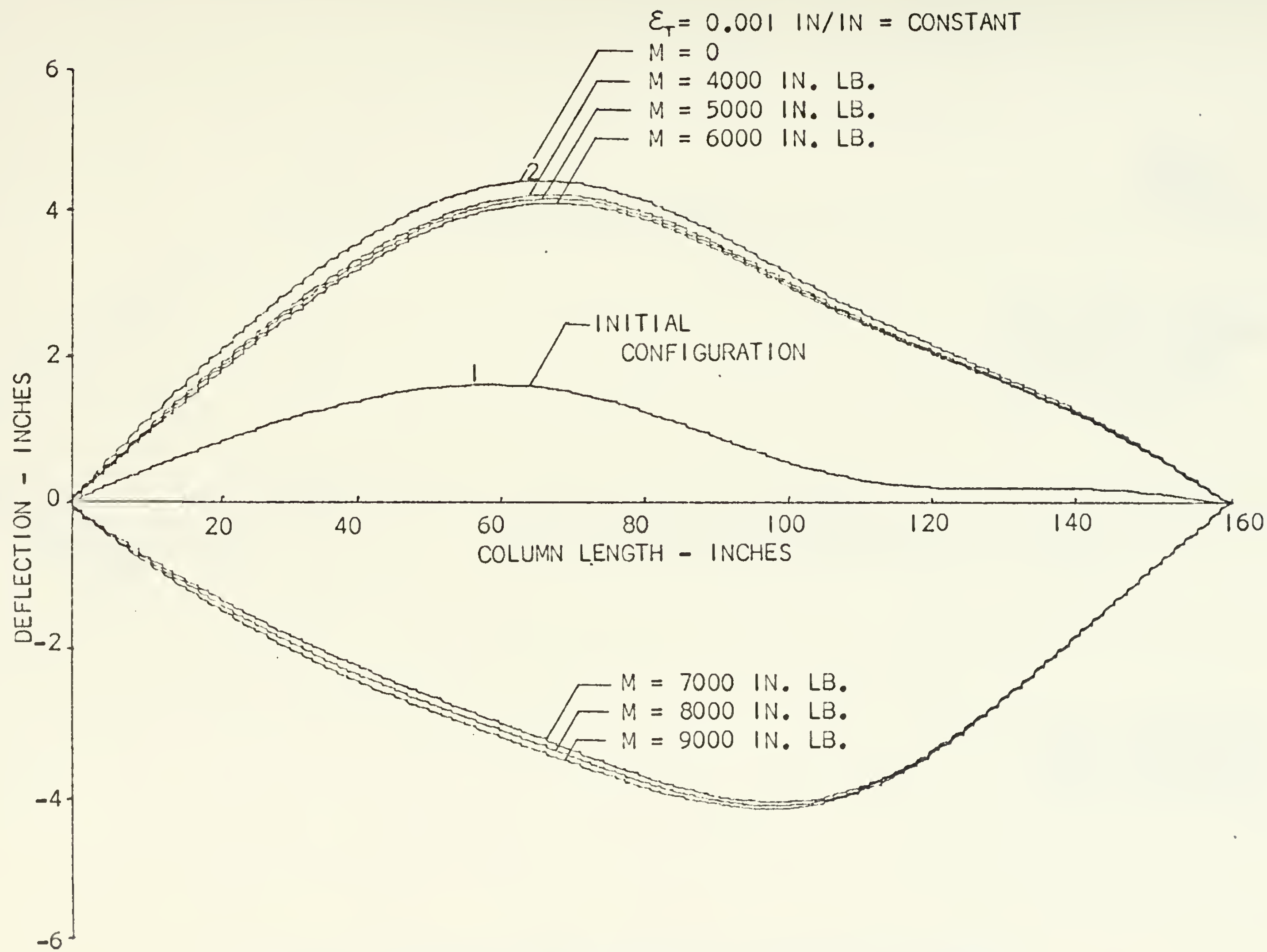


FIGURE 6. SNAP-THROUGH PHENOMENON IN SINGLE-SPAN COLUMN

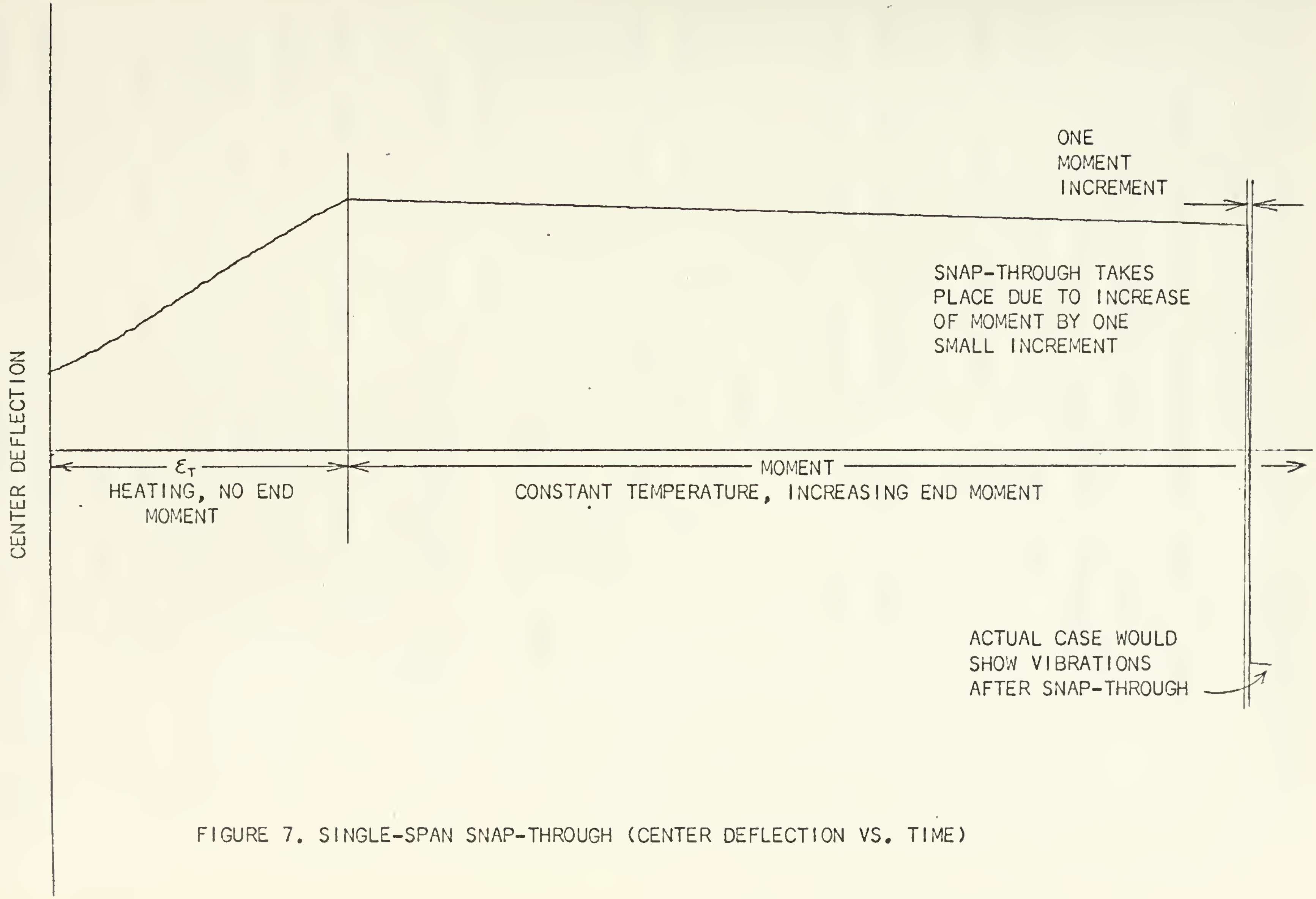


FIGURE 7. SINGLE-SPAN SNAP-THROUGH (CENTER DEFLECTION VS. TIME)

end moment, and then thermal strain is held constant and the end moment is increased at a uniform rate.

It may be noted that a similar analysis could be made for a column under lateral loading instead of end moment loading.

It will be of interest to explain why the phenomenon of snap-through is considered to be of great importance. In a piping system which is heated up, considerable energy is stored in the form of strain energy. If snap-through or a similar phenomenon could take place, some of this energy might be suddenly released and have a catastrophic effect on the piping system or adjacent installations. In the case of a single-span straight-line system, if the ends should be pinned, so that no moments may be applied, it is clear that snap-through cannot take place. If the ends are fixed to heavy equipment, such as pumps or vessels, the restraining influence of these items would indeed result in end moments being applied to the pipe, but it is difficult to envision these moments actually resulting in snap-through.

However, in the case of multi-span straight-line configurations, at the intermediate supports adjacent spans exert moments on each other, and it is not immediately clear that as the system is heated, perhaps nonuniformly, these intermediate moments might not increase in such a way as to cause snap-through. The intuitive feeling that this would not take place was insufficient basis for asserting that it could not happen. Accordingly, considerable attention was focused on this possibility. It was only after studying many different initial configurations and thermal loading sequences that a reasonable under-standing was achieved of the restraining influence which one span has

upon an adjacent span and of how this serves to prevent the phenomenon of snap-through, even if one end of a multi-span configuration is subjected to increasing external moment.

Clearly, the simplest multi-span configuration is that having two-spans. Accordingly, this case was studied in detail and computer programs were written for its analysis. One of the cases which was studied is illustrated in Fig. 8. In this case a negative moment of increasing magnitude was applied at the extreme left causing the column to go through the succession of configurations shown. (Incidentally, although initially there was axial compression in the column due to initial heating, by the time the last configuration shown was obtained, the axial load in the left span had become tensile.)

This example provided the insight to see why snap-through, at least in any sense similar to that for the single-span column, is not possible for two-span columns. The original, incorrect, view was that moment loading at the left end of a two-span column was, in effect, simply a way of applying moment to the left end of the right-hand span; increasing the leftmost moment sufficiently would eventually apply a very large moment, at the junction point, to the left end of the right span. This is true enough, but the continuity between left and right spans does not permit the latter to snap-through as would be the case if it were the only span and the junction moment were directly applied at its left end. In effect, the left span acts as a stabilizing influence to prevent snap-through of the right span.

The situation may be visualized more clearly by imagining that the continuity between the two spans is "one-way" in the sense of

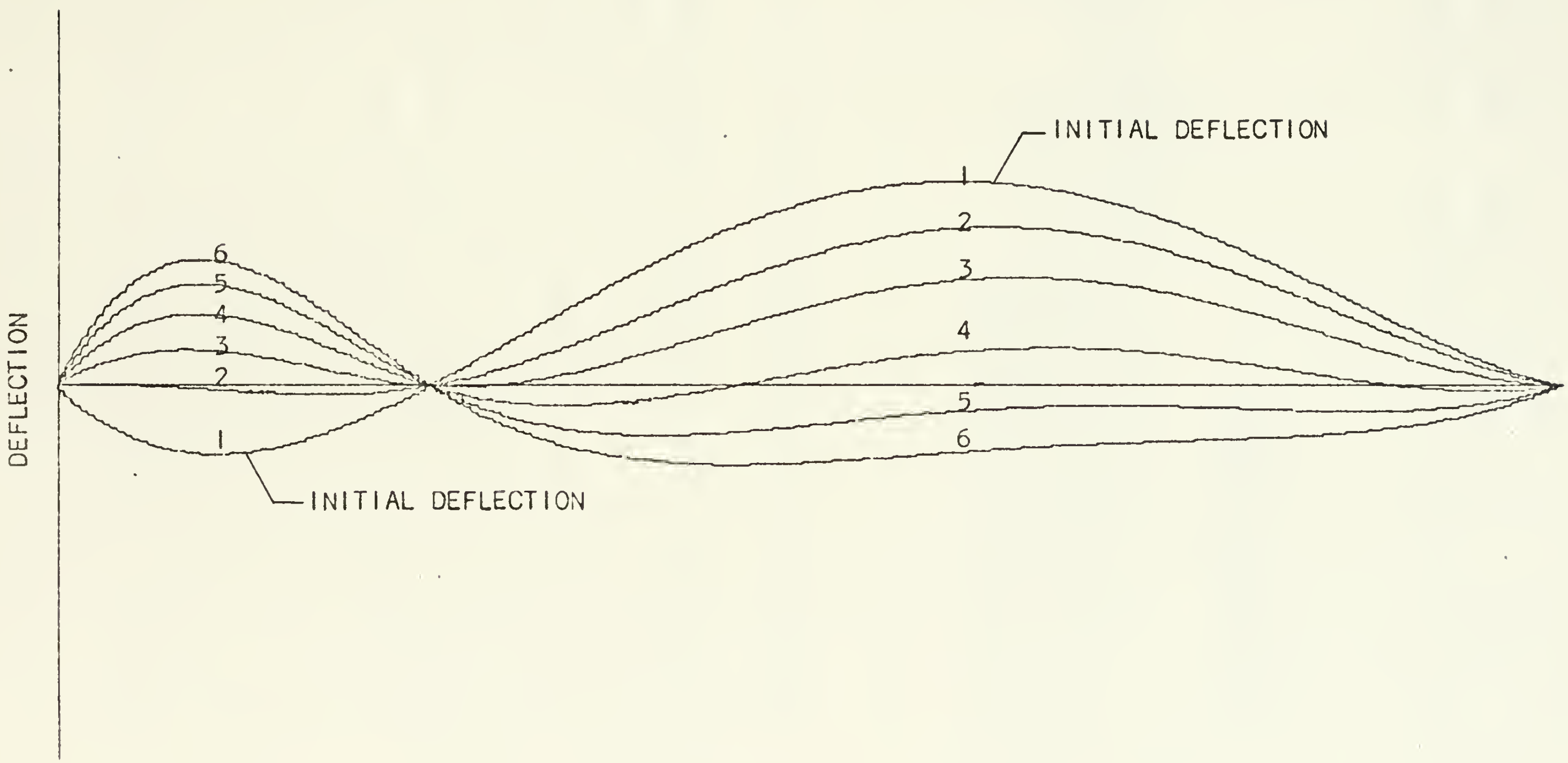


FIGURE 8. TWO-SPAN COLUMN WITH VARYING LEFT END MOMENT

the mechanism shown in Fig. 9. If the two spans were joined in this manner, snap-through of the right span would indeed take place when the junction moment reached a critical value. However, with the true continuity of a continuous column such behavior can not take place.

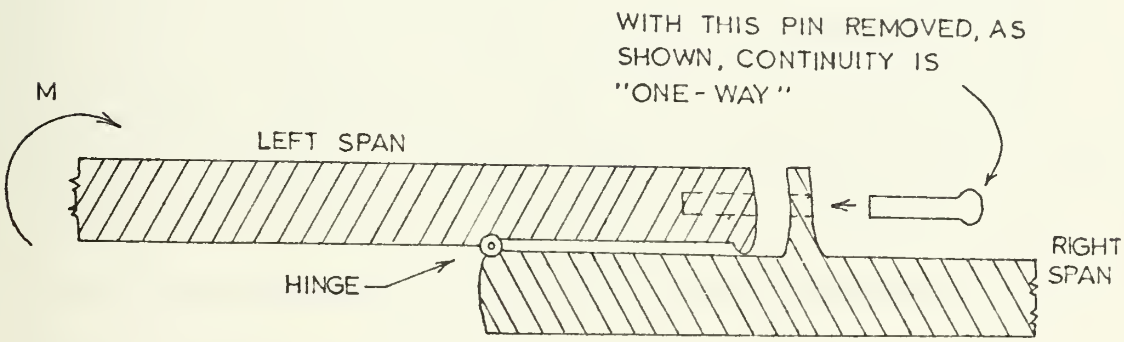


Figure 9. "One-way" continuity of two-span junction

Attempts to create a situation in which snap-through takes place in a two-span column have proved unsuccessful. Study of the behavior in these cases has added strength to the conviction that such a phenomenon cannot take place, even under the action of increasing externally applied moments at the ends or at the intermediate supports. However, this growing conviction cannot be understood as a convincing demonstration that the phenomenon cannot indeed take place, perhaps only for certain unfortunate combinations of physical parameters. Accordingly, further study of snap-through in multi-span columns is suggested as an appropriate topic for further investigation.

B. FOURIER REPRESENTATION OF THE CONFIGURATIONS

A question arises of the ability of Fourier sine series, as employed herein, to represent non-zero curvature at the ends of spans where each sine term vanishes. Certainly, the value of curvature given by such a series must be zero when x=0 and x=L. If we have initial deflections which have non-zero curvature at the ends of the span (or spans), our Fourier series must be able to represent this adequately. Also, if we have non-zero moment at these points (due to the application of external moments or due to enforcing an intermediate constraint) this corresponds to non-zero curvature change, which must be adequately represented by our Fourier series.

Accordingly, actual plots of curvature, as determined from the Fourier series, were made in several cases where the value of curvature could be determined by other means. Figures 10, 11, and 12 show a comparison of curvature in a case where the initial configuration (in the sense of Fig. 5) had uniform negative curvature and where the intermediate constraint produced a piece-wise linear moment change (i.e., curvature) that was zero at each end and a maximum at the constraint. The final curvature is the superposition of these, consisting of two straight segments rising from -.001332 to .000666 and back down to -.001332. (In Figs. 10, 11, and 12 these straight line segments show a very small waviness due to the method of plotting.) These figures also show the Fourier series representation of curvature, using 25, 50, and 100 terms, respectively. It is seen that although indeed the Fourier representation of curvature does vanish at the constraint points, this

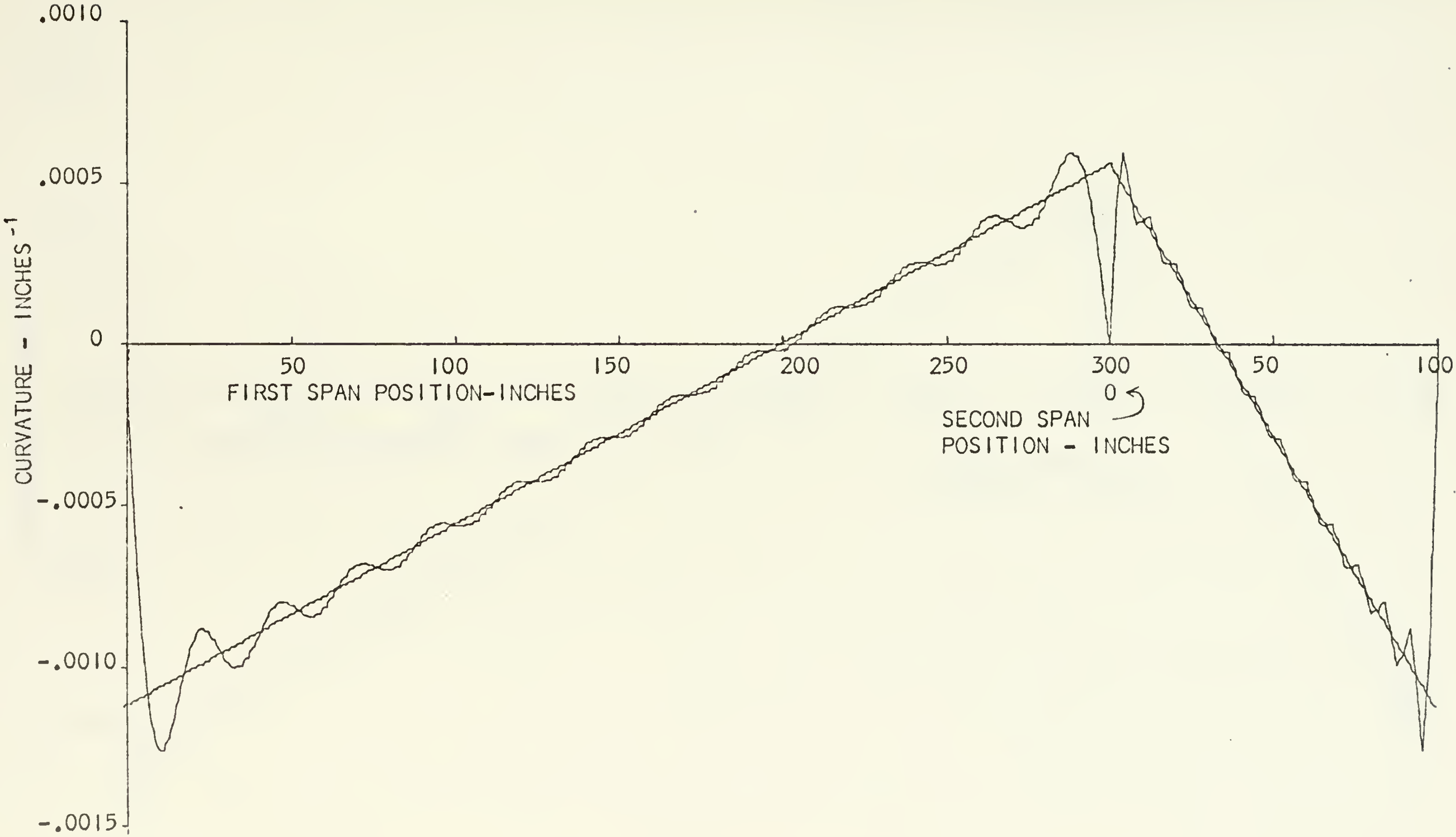


FIGURE 10. CURVATURE IN TWO-SPAN COLUMN, 25 FOURIER TERMS

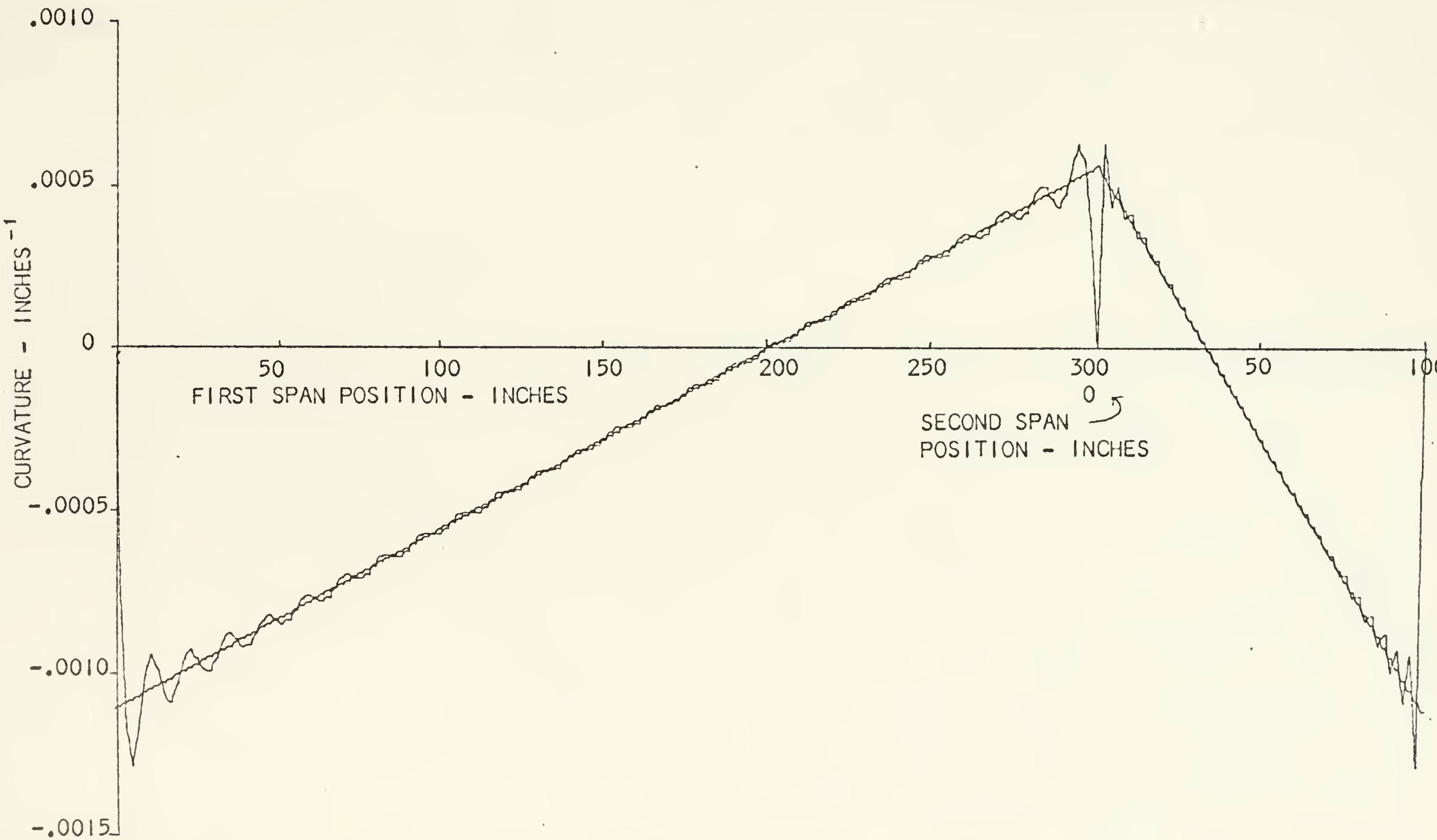


FIGURE 11. CURVATURE IN TWO-SPAN COLUMN, 50 FOURIER TERMS

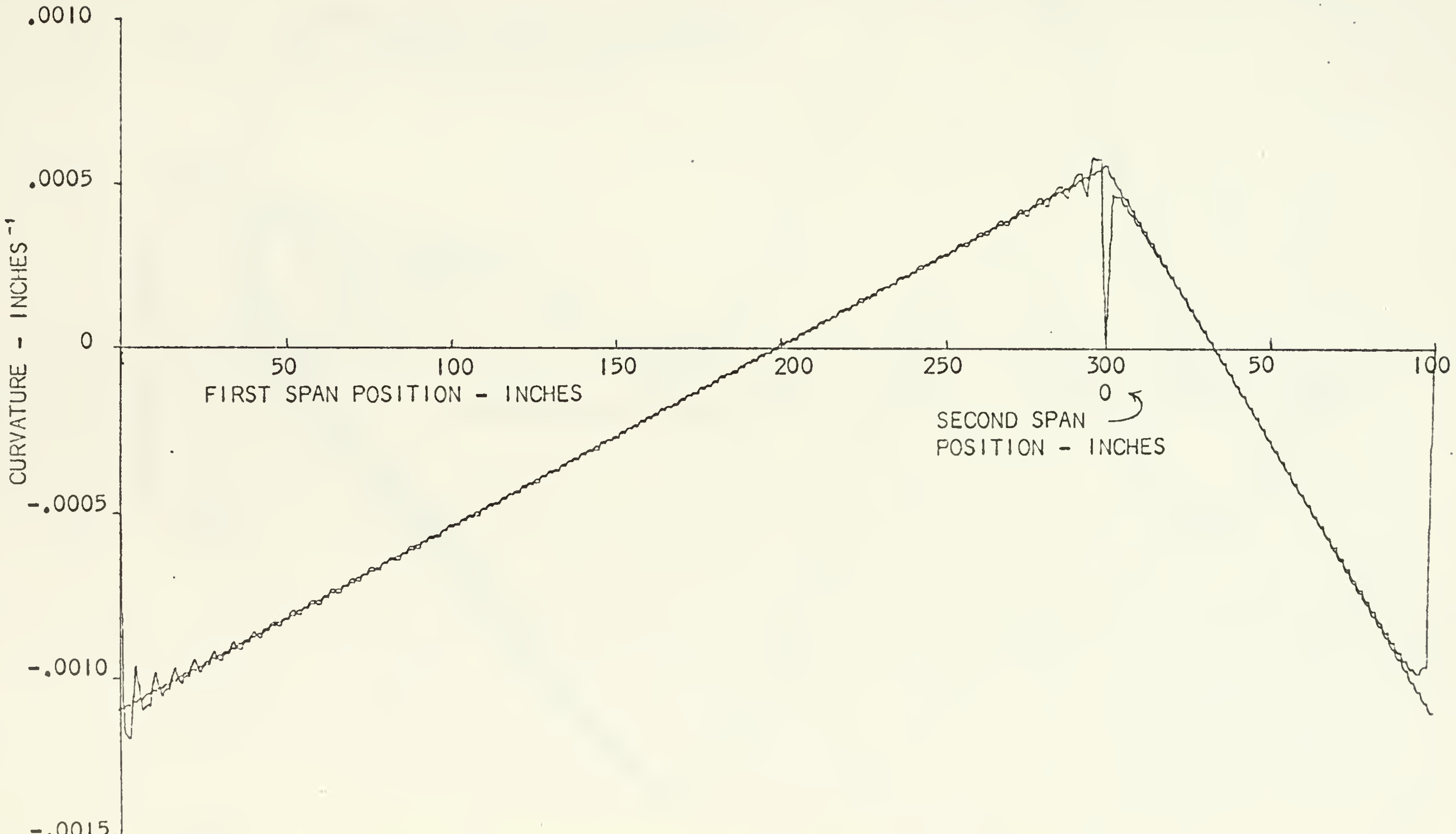


FIGURE 12. CURVATURE IN TWO-SPAN COLUMN, 100 FOURIER TERMS

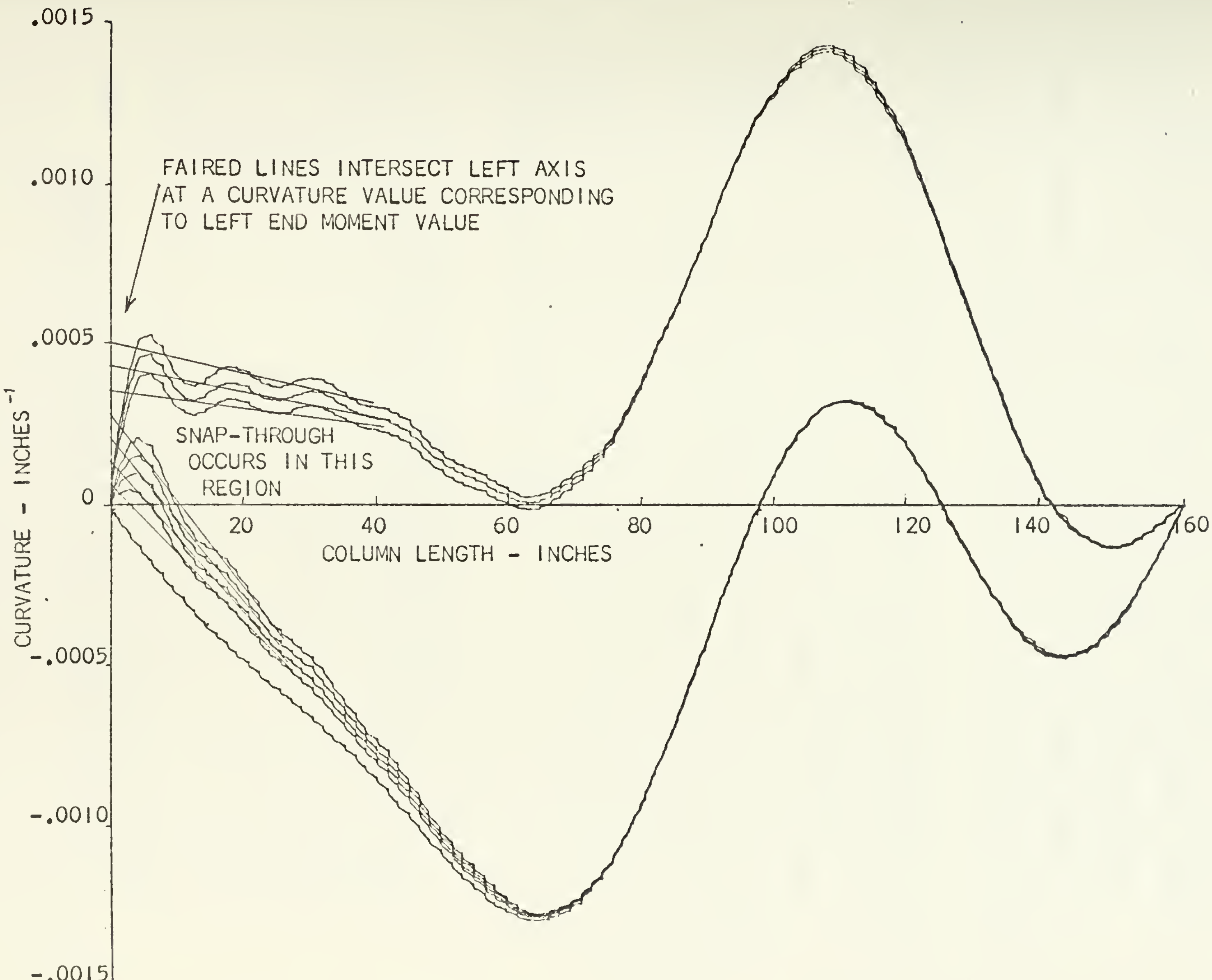


FIGURE 13. CURVATURE IN SNAP-THROUGH OF SINGLE-SPAN COLUMN

is a very local phenomenon and elsewhere the Fourier series provides a good representation of the correct values.

Another example is shown in Fig. 13 where values of curvature as represented by Fourier series are plotted for different values of left end moment in a case of snap-through. Fairing straight lines through the left ends of the curves gives intersections with the left vertical axis which correspond to the actual moments applied at the left.

Probably the most convincing verification of the adequacy of the Fourier representation is the relative insensitivity of significant results (in a particular problem) to the number of Fourier terms employed. In a particular case of snap-through, Table I below shows that snap-through occurred in the same moment interval for all values of n down to as low as 3. It is interesting to note in Table II that, even for n=1, the axial load obtained is in very good agreement with the load obtained using 100 terms.

Table I. Values of end moment between which snap-through occurred, for different values of n.

Moment (in.lb.)	n=100	n=50	n=25	n=10	n=5	n=3	n=2	n=1
60450								
	•	•	•	•	•	•		
60460								
							•	•
60470								

Note: Dots indicate the moment interval in which snap-through occurred. The tabulation is for a particular problem described on page 31.

Table II. Value of axial load in pounds for different values of n and M.

Moment (in.lb.)	60450	60460	60470
n=100	38489.000514	38491.906680	38489.899526
n=50	38489.000515	38491.906680	38489.899526
n=25	38489.000517	38491.906680	38489.899527
n=10	38489.000550	38491.906674	38489.899548
n=5	38489.000764	38491.906634	38489.899684
n=3	38489.009402	38491.905053	38489.905163
n=2	38489.011286	38491.009623	38489.906358
n=1	38489.011298	38491.009624	38489.906349

The tabulation is for a particular problem described below.

Due to computational difficulties in the vicinity of snap-through, values of moment taken at smaller than 10 inch-pound increments provided somewhat erratic results.

The results tabulated in Tables I and II are for a steel pipe whose cross sectional area is 2.945 in^2. The length is 200 inches, the moment of inertia of the cross section is 5.2 in^4, and Young's modulus of elasticity is 30 x 10^{6}psi. Moment is applied at only one end of the span. The initial configuration was similar to that shown in Fig. 6.

IV. ANALYSIS OF A TYPICAL TWO-SPAN PROBLEM

Using the program described in Appendix A, configurations resulting from the application of many different combinations of end moments and strain on a two-span column can be obtained. The following cases illustrate several of the ways in which the program may be utilized. These examples involve a two-span steel pipe with cross sectional area of 2.945 in^2. The left span is 100 inches long and the right span 300 inches. The moment of inertia is 5.2 in^4 and Young's modulus of elasticity is 30×10^6 psi. It should be noted, however, that a column of any constant cross section or material could have been used.

Case I - As illustrated in Fig. 14, the pipe is subjected to thermal strain only. Curve 1 shows the initial configuration. Subsequent numbered curves represent the configuration corresponding to increasing values of thermal strain.

Case II - Here the pipe is subjected only to an increasing moment. Figure 15 shows the sequence of resulting configurations.

Case III - This example shows the resulting configuration of the pipe (see Fig. 16) with simultaneously increasing thermal strain and left end moment.

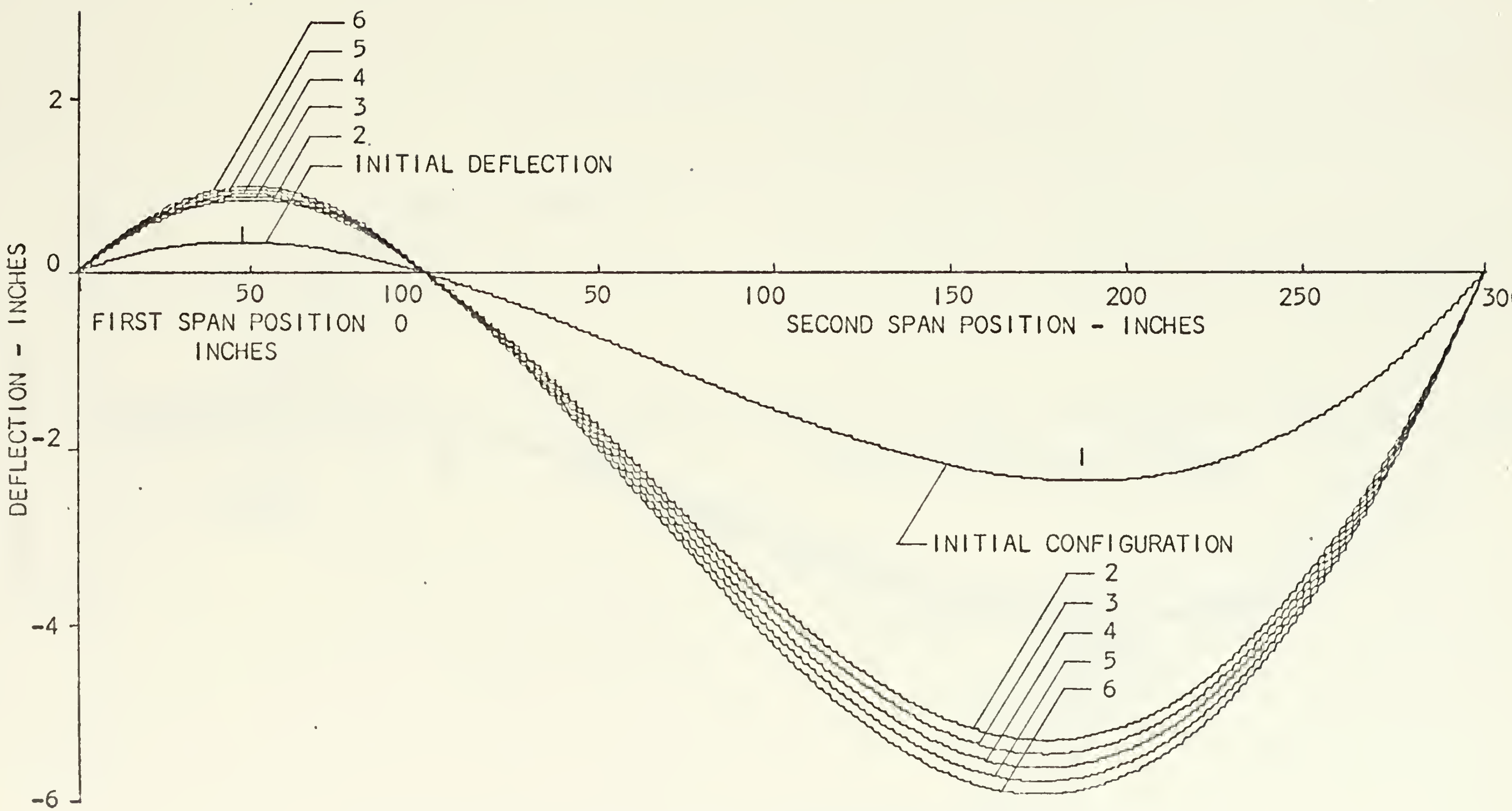


FIGURE 14. TWO-SPAN COLUMN WITH INCREASING THERMAL STRAIN

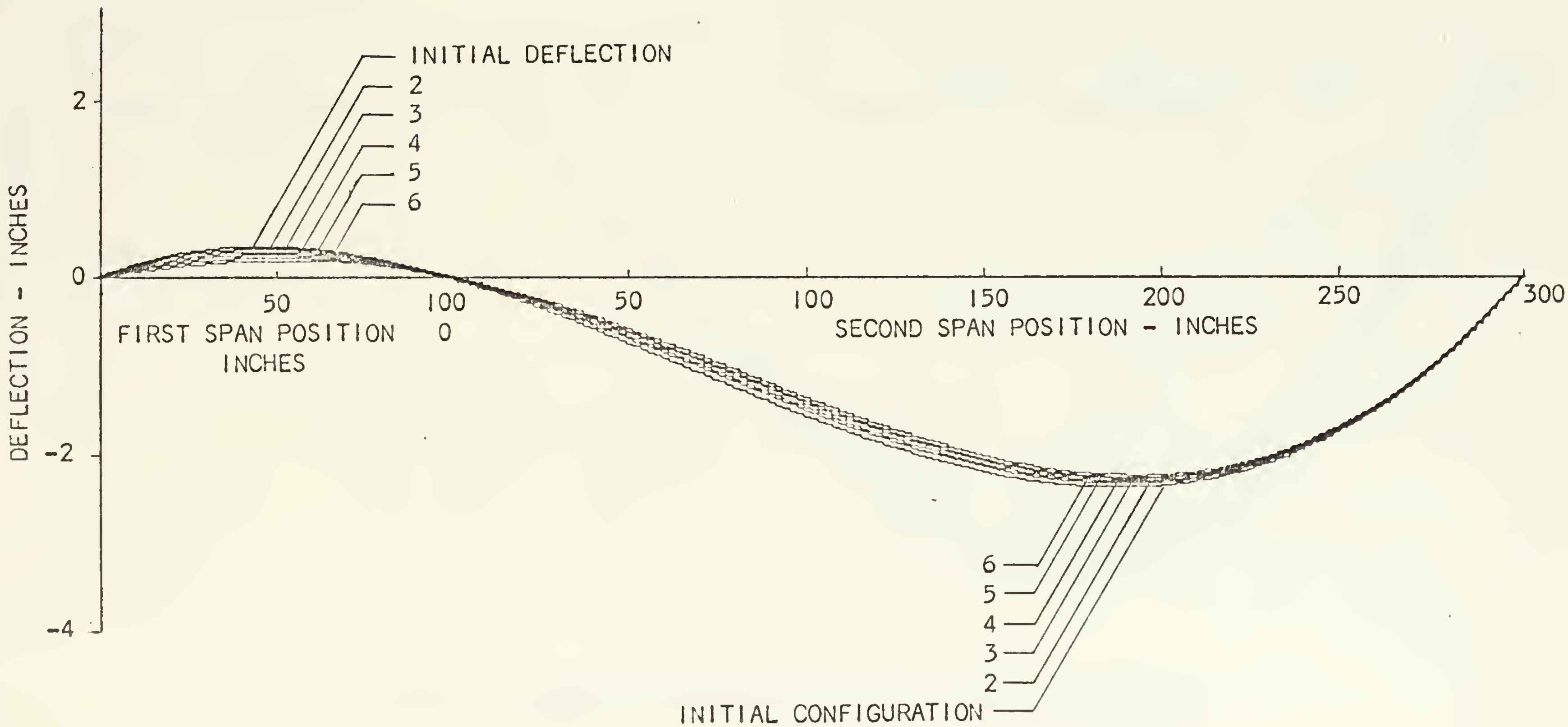


FIGURE 15. TWO-SPAN COLUMN WITH INCREASING LEFT END MOMENT

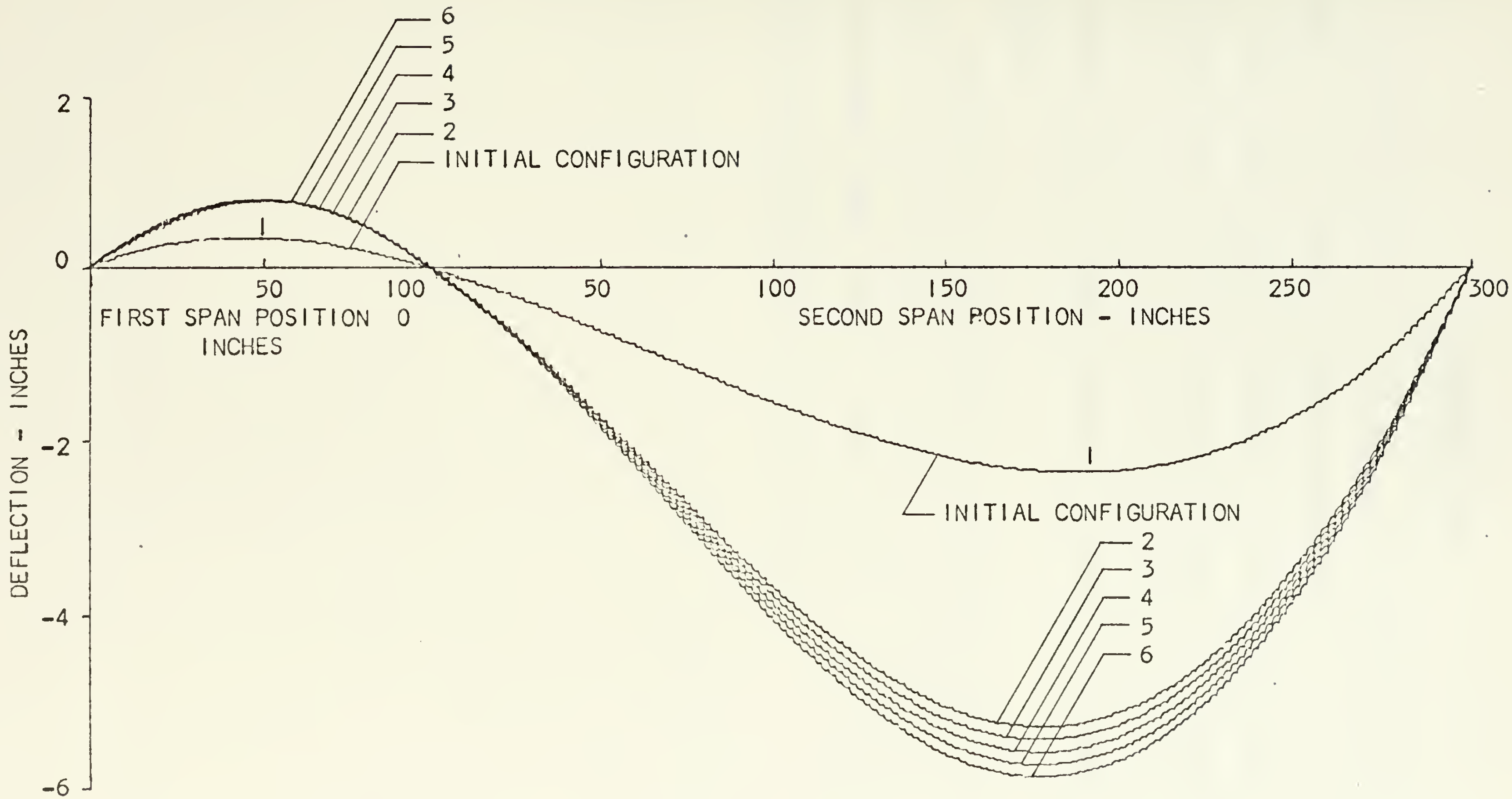


FIGURE 16. TWO-SPAN COLUMN WITH SIMULTANEOUSLY INCREASING THERMAL STRAIN AND LEFT END MOMENT

V. RECOMMENDATIONS

If future studies are made using the analysis set forth in this thesis, the following areas are recommended.

The problem should be extended to include multi-span columns. In the analysis of two or more spans it would be convenient to have a better way of obtaining the junction moment or moments. Perhaps a method can be developed that does not require an initial approximation.

Another area of study would be to develop a proof that snap-through cannot take place or develop a clear delineation of the circumstances under which it could take place. The study probably should focus initially on two-span columns.

APPENDIX A - DESCRIPTION OF COMPUTER PROGRAM

A. GENERAL REMARKS

The digital computer program used to carry out the analysis set forth in this thesis is a double precision FORTRAN language program. It is self contained except for one outside subroutine used to draw the configurations. This subroutine or a similar one should be available in any computer library. Various write statements are interspersed throughout the program to allow the user to follow the progress of the computations, and to help trace any errors. These statements may of course be left out of the program to lessen the amount of unneeded output.

B. BASIC FUNCTIONS PERFORMED BY THE PROGRAM

The basic program for the single-span plots the initial deflection, determines the axial load on the column, and plots the final deflection. The axial load is determined using a Newton Raphson technique on Eq. (19). Iteration continues until convergence is reached.

For the two-span problem, a process is employed of repeatedly selecting values of the junction moment. Then the two spans are analyzed and the two junction slopes are computed and compared. Then the correct value of the junction moment is found using the method of successive decimation. Although the slope intercept method is more accurate when it works, its unreliability causes it to be less desirable for use in the general program. After the junction moment is found, the final deflection is then plotted.

C. INITIAL DEFLECTION

The Fourier coefficients for the initial deflection of the column are computed for the analysis previously described. The program accepts coefficients describing a polynomial curve which passes through the left support of span one and the right support of span two. Then the modification of this curve is computed which enforces the intermediate constraint; this modified curve is also in the form of a polynomial. This yields two polynomial curves, one for each span; cf. Eqs. (27) and (28). Then the Fourier coefficients a_n are evaluated for each span.

1. Subroutine SINCOE

This subroutine evaluates the integral

$$GA(m,n) = \int_0^{\pi} y^m \sin\left(ny\right) dy \tag{A.1}$$

which is used in the evaluation of the Fourier coefficients. The method involves evaluating integral number 6, page 117, of Ref. [4] . See program listing in Appendix C.

2. Evaluation of Fourier Coefficients

The Fourier coefficients are evaluated according to the following formula:

$$a_n = \frac{2}{L} \int_0^L g(x) \sin\left(\frac{n\pi x}{L}\right) dx, \; n = 1,2,\ldots, \tag{A.2}$$

where L is the length of the span and n is the number of coefficients desired. For the left span, y, as given by Eq. (27), is substituted for the value of g(x), and for the right span Eq. (28) is used.

The method carried out in the program can best be illustrated through the use of an example. Suppose the initial configuration f(x) is given by the equation

$$f(x) = Ax^2 + Bx. \tag{A.3}$$

Then Eqs. (27) and (28) become

$$y = Ax^2 + Bx - \frac{WL_2}{6EI(L_1+L_2)} \times \left[(L_1+L_2)^2 - x^2 - L_2{}^2 \right] \tag{A.4}$$

and

$$y = A(x^*+L_1)^2 + B(x^*+L_1) - \frac{WL_2}{6EI(L_1+L_2)} (x^*+L_1) \left[(L_1+L_2)^2 - (x^*+L_1)^2 - L_2{}^2 \right.$$

$$\left. + \frac{(L_1+L_2)}{L_2(x^*+L_1)} x^{*3} \right] \tag{A.5}$$

respectively. Equation (A.4) can be rewritten as

$$y = Ax^2 + Bx - Cx + Dx^3 \tag{A.6}$$

and Eq. (A.5) can be rewritten as

$$y = A(x^*+L_1)^2 + B(x^*+L_1) - C(x^*+L_1) + D(x^*+L_1)^3 - Fx^{*3}, \tag{A.7}$$

where

$$C = \frac{WL_2(L_1+2L_1L_2)}{6EI(L_1+L_2)}, \tag{A.8}$$

$$D = \frac{WL_2}{6EI(L_1+L_2)}, \tag{A.9}$$

and

$$F = \frac{W}{6EI}. \tag{A.10}$$

For the first span, Eq. (A.2) becomes

$$a_n = \frac{2}{L_1} \left[A \int_0^{L_1} x^2 \sin\left(\frac{n\pi x}{L_1}\right) dx + B \int_0^{L_1} x \sin\left(\frac{n\pi x}{L_1}\right) dx \right.$$

$$\left. - C \int_0^{L_1} x \sin\left(\frac{n\pi x}{L_1}\right) dx + D \int_0^{L_1} x^3 \sin\left(\frac{n\pi x}{L_1}\right) dx \right]. \tag{A.11}$$

The integrals are all of the form

$$\int_0^{L_1} x^m \sin\left(\frac{n\pi x}{L_1}\right) dx . \tag{A.12}$$

Making the substitution $y = \frac{\pi x}{L_1}$, this integral may be put in the form

$$\left(\frac{L_1}{\pi}\right)^{m+1} \int_0^{\pi} y^m \sin\left(ny\right)dy = \rho^{n+1}\ GA(m,n), \tag{A.13}$$

where

$$\rho = \frac{L_1}{\pi}, \tag{A.14}$$

and the definition, (A.1), is used. Thus Eq. (A.11) can be rewritten in the form

$$a_n = \frac{2}{L_1}\left[A\rho^3 GA(2,n) + B\rho^2 GA(1,n) - C\rho^2 GA(1,n) + D\rho^4 GA(3,n)\right]$$

$$= \frac{2}{L_1} \Sigma\ A_i\ \rho^{i+1}\ GA(i,n) \tag{A.15}$$

Note, from Eqs. (27) and (28), that the maximum degree of the adjusted polynomials is Max(3,m^*), where m^* is the degree of f(x). The number n of Fourier coefficients to be employed, and the integer m^* are among the inputs to the program. Internally, m^* is increased, if necessary, to a minimum value of 3. Then SINCOE is used to generate the numerical constants GA(i,j), i = 1,2, ..., m^*; j = 1,2, ..., n. Thus, the coefficients are available for the evaluation of the Fourier coefficients as represented by such equations as (A.15), the coefficients A, B, C, ..., in which are obtained in an obvious manner.

The Fourier coefficients for the second span are found in basically the same manner, except that now the computations are more numerous due to the necessity of expanding powers of (x^*+L). It can be seen that for the second span Eq. (A.2) becomes quite lengthy since g(x) will contain many terms. Looking only at the first term in the example, we have, after expansion

$$a_n = \frac{2}{L_2}\left\{A\left[\int_0^{L_2} x^{*2}\sin\left(\frac{n\pi x}{L_2}\right)dx + L_1\int_0^{L_2} x^{*}\sin\left(\frac{n\pi x}{L_2}\right)dx\right.\right.$$

$$\left.\left. + L_1^{\,2}\int_0^{L_2}\sin\left(\frac{n\pi x}{L_2}\right)dx\right] + \ldots\right\} \quad . \qquad (A.16)$$

This can also be written in the form

$$a_n = \frac{2}{L_2}\left\{A\ \rho^2\left[GA(2,n) + L_1\rho^2 GA(1,n) + L_1^{\,2}\rho GA(0,n)\right] + \ldots\right\} \qquad (A.17)$$

where now

$$\rho = \frac{L_2}{\pi} \ .$$

D. VARIATIONS IN THE PROGRAM

Different combinations of strain and end moments to be applied to the column can be incorporated into the program. This can be accomplished easily through the use of the input cards described below. The program can also be altered through an additional DO-loop to show a "time history" of the column. The strain and/or end moments can be incremented in the DO-loop so that intermediate configurations may be plotted. This is useful in looking at snap-through. The program listed in Appendix C has this variation incorporated.

E. USE OF THE PROGRAM

This program may be used by inserting the following 21 input cards in the designated places at the beginning of the program. Each card contains only one entry. Note that these cards are made a part of the program deck and are not read in as data following the program. See program listing for example. All cards must be punched beginning in column seven.

DIMENSION GA(MPOL,NFOR) - integer numbers must be inserted for MPOL and NFOR

MPOL = maximum degree of initial deflection function

NFOR = number of Fourier coefficients desired

NDEC = number of decimations in determining junction moment

C6 = initial function coefficient

C5 = initial function coefficient

C4 = initial function coefficient

C3 = initial function coefficient

C2 = initial function coefficient

C1 = initial function coefficient

L1 = length of left span

L2 = length of right span

E = Young's modulus of elasticity

W = cross section area of the column in inches^2

Z = moment of inertia of the column in inches^4

EPS1 = applied strain in left span in inches/inch

EPS2 = applied strain in right span in in/in

SS = applied left end moment in pound inches

UU = applied right end moment in pound inches

XMOMT = starting moment for successive decimation iteration

DELTA = initial increment for successive decimation iteration

Notes:

1. All inputs are floating point numbers (D format) except MPOL and NFOR, which are integer numbers.

2. There must be an input card for each input listed, even if it is zero.

3. Each initial function coefficient must correspond to the power of x it is associated with. That is

$$f(x) = C6x^6 + C5x^5 + C4x^4 + C3x^3 + C2x^2 + C1x$$

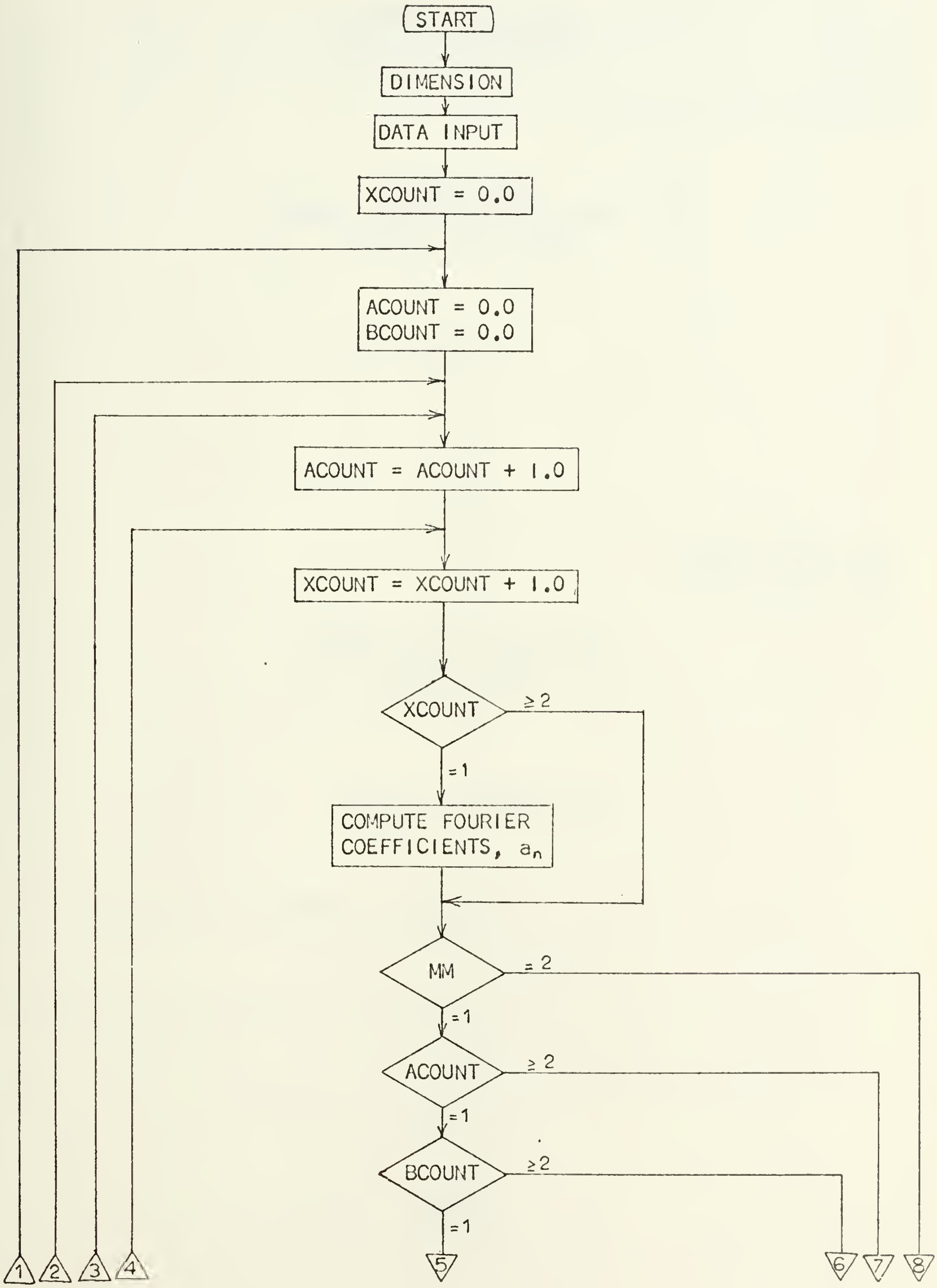
(A polynomial of degree greater than six can be accommodated only by making internal changes in the program: these should be rather obvious.)

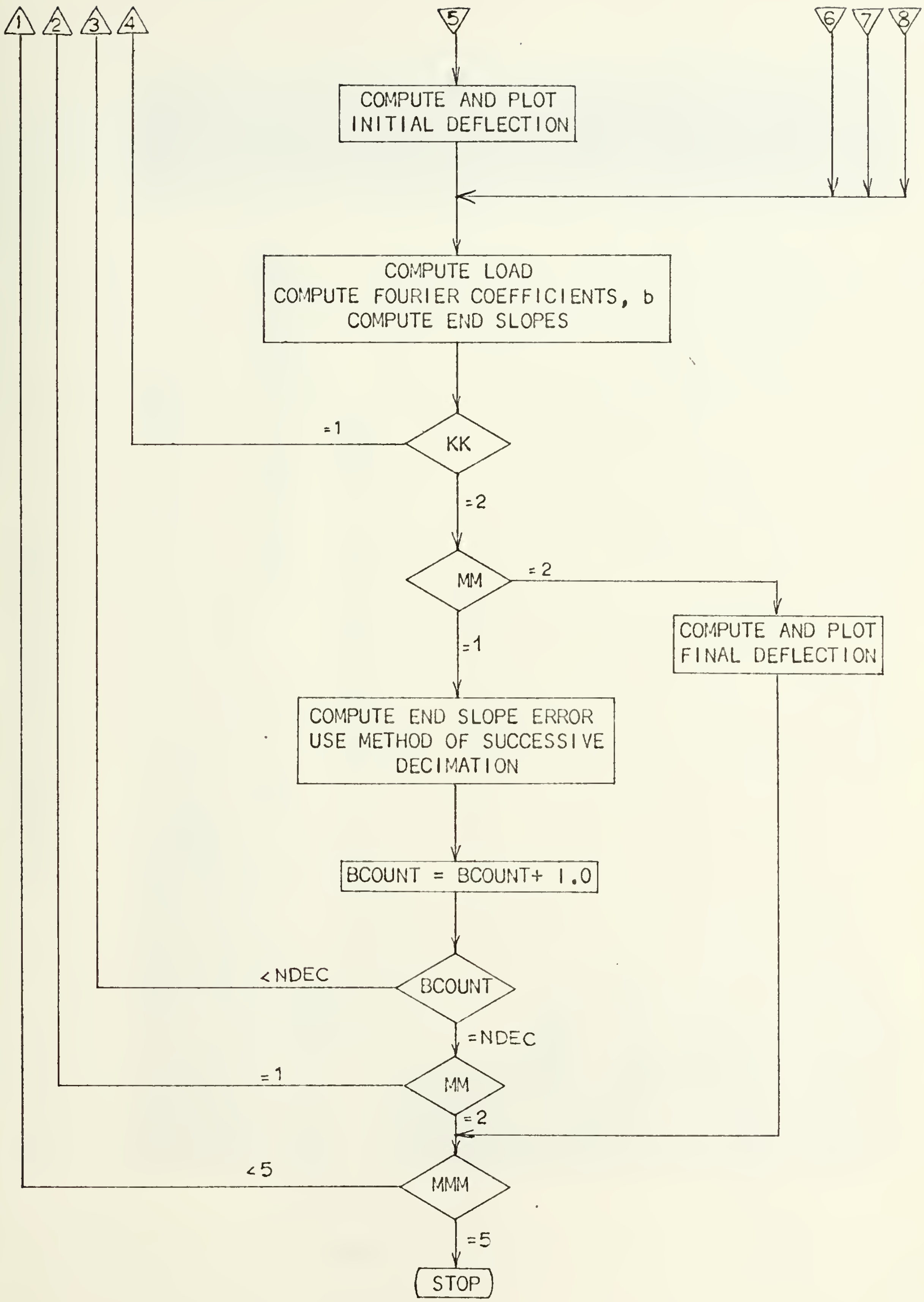
4. The program is dimensioned for a maximum number of 50 Fourier coefficients and a maximum combined span length of 400 inches. (This limitation is related to the fact that deflections are computed at two-inch intervals and are stored prior to output.) If the user wishes to extend these limits, the dimension statements must be altered accordingly.

5. Likewise the call statement for the DRAW subroutine may have to be altered to accommodate the particular problem in question. (For example, to provide a desired scaling.)

6. Where to start the iteration for the method of successive decimation, i.e., selection of the input XMOMT, poses a problem. Experience and perhaps a trial run will best produce a suitable starting point.

APPENDIX B - FLOW CHART




1
2
3
4
5
6
7
8
COMPUTE AND PLOT INITIAL DEFLECTION
COMPUTE LOAD
COMPUTE FOURIER COEFFICIENTS, b
COMPUTE END SLOPES
KK
=1
=2
MM
=2
COMPUTE AND PLOT FINAL DEFLECTION
=1
COMPUTE END SLOPE ERROR
USE METHOD OF SUCCESSIVE DECIMATION
BCOUNT = BCOUNT+ 1.0
BCOUNT
<NDEC
=NDEC
MM
=1
=2
MMM
<5
=5
STOP

APPENDIX C - PROGRAM LISTING

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(51),B(51),C(51),H(51),HC(51),X(202),Y(202)
      DIMENSION THETER(2),THETEL(2)
      DIMENSION XC(202),YC(202)
      DIMENSION XN(202),YN(202),XM(202),YM(202)
      DIMENSION A1(51),A2(51)
      REAL*8 LABEL/8H        /
      REAL*8 ITITLE(12)/'ROB PLOE','GER     ','TWO SPAN',' COLUMN',8*'
     1     '/
      REAL*4 XA(202)
      REAL*4 YA(202)
C
C           THE FOLLOWING 21 CARDS ARE NECESSARY INPUTS
C
      DIMENSION GA(4,25)
      MPOL=4
      NFOR=25
      NDEC=4
      C6=0.D0
      C5=0.D0
      C4=2.D-9
      C3=-1.9D-6
      C2=5.4625D-4
      C1=-4.25D-2
      L1=100
      L2=300
      E=30000000.0D0
      W=2.945D0
      Z=5.20D0
      EPS1=.0001D0
      EPS2=.00005D0
      SS=-200.D3
      UU=0.D0
      XMOMT=36.D3
      DELTA=-1.D3
C
      XCOUNT=0.D0
      PI=4.D0*DATAN(1.D0)
      ESP=1.D-5
      DO 10 I=1,NFOR
      A(I)=0.0D0
      A1(I)=0.D0
```

```
      A2(I)=0.D0
   10 B(I)=0.0D0
C
C             THE FOLLOWING DO LOOP ALLOWS 'TIME HISTORY' TO BE SHOWN
C
      DO 99 MMM=1,5
      NG=2
      IF(MMM.EQ.5) NG=3
      ERRO=10.D0
      ACOUNT=0.D0
      BCOUNT=0.D0
      XMOM=XMOMT
      DELTF=DELTA
C
C             THE FIRST TIME THROUGH THE FOLLOWING DO LOOP THE PROBLEM
C             IS WORKED AND JUNCTION MOMENT DETERMINED.  THE SECOND
C             TIME THROUGH THE FINAL DEFLECTION IS CALCULATED AND
C             PLOTTED
C
      DO 90 MM=1,2
      IF(MM.EQ.2) GO TO 91
   80 DELT=DELTF
      ACOUNT=ACOUNT+1.D0
      WRITE(6,1110) XMOM
 1110 FORMAT('0',10X,'MOMENT GUESS=',F10.1)
   91 CONTINUE
C
C             THE FOLLOWING DO LOOP CAUSES LEFT SPAN TO BE ANALIZED
C             FIRST, THEN RIGHT SPAN
C
      DO 22 KK=1,2
      IF(KK.EQ.1) L=L1
      IF(KK.EQ.2) L=L2
      IF(KK.EQ.1) EPS=EPS1
      IF(KK.EQ.2) EPS=EPS2
      PC=PI**2*E*Z/L**2
      IF(KK.EQ.1) S=SS
      IF(KK.EQ.1) U=XMOM
      IF(KK.EQ.2) S=XMOM
      IF(KK.EQ.2) U=UU
      XCOUNT=XCOUNT+1.D0
      IF(XCOUNT.GE.2) GO TO 57
      IF(MPOL.LT.3) MPOL=3
      EL1=L1
      EL2=L2
C
C             COMPUTE AND STORE SINCOE INTEGRALS
C
```

```
      DO 21 M=1,MPOL
      DO 19 N=1,NFOR
      CALL SINCOE(M,N,R)
      GA(M,N)=R
   19 CONTINUE
   21 CONTINUE
C
C             COMPUTE FOURIER COEFFICIENTS
C
      DEL=C6*EL1**6+C5*EL1**5+C4*EL1**4+C3*EL1**3+C2*EL1**2+C1*EL1
      EP=(3.DO*(EL1+EL2)*E*Z*DEL)/(EL1**2*EL2**2)
      FC=EP*EL2*(EL1**2+2.DO*EL1*EL2)/(6.DO*(EL1+EL2)*E*Z)
      FD=EP*EL2/(6.DO*(EL1+EL2)*E*Z)
      FE=EP/(6.DO*E*Z)
      T=EL1/PI
      TT=EL2/PI
      POL6=0.DO
      POLL6=0.DO
      POL5=0.DO
      POLL5=0.DO
      POL4=0.DO
      POLL4=0.DO
      DO 55 N=1,NFOR
      EN=N
      GG=-1.DO/EN*(-1.DO)**N+1.DO/EN
      IF(MPOL.EQ.5) GO TO 1
      IF(MPOL.EQ.4) GO TO 2
      IF(MPOL.EQ.3) GO TO 3
      POL6=T**7*GA(6,N)
      POLL6=TT**7*GA(6,N)+6.DO*EL1*TT**6*GA(5,N)+15.DO*EL1**2*TT**5
     1*CA(4,N)+20.DO*EL1**3*TT**4*GA(3,N)+15.DO*EL1**4*TT**3*GA(2,N)
     2+6.DO*EL1**5*TT**2*GA(1,N)+EL1**6*TT*GG
    1 POL5=T**6*GA(5,N)
      POLL5=TT**6*GA(5,N)+5.DO*EL1*TT**5*GA(4,N)+10.DO*EL1**2*TT**4
     1*GA(3,N)+10.DO*EL1**3*TT**3*GA(2,N)+5.DO*EL1**4*TT**2*GA(1,N)
     1+EL1**5*TT*GG
    2 POL4=T**5*GA(4,N)
      POLL4=TT**5*GA(4,N)+4.DO*EL1*TT**4*GA(3,N)+6.DO*EL1**2*TT**3
     1*GA(2,N)+4.DO*EL1**3*TT**2*GA(1,N)+EL1**4*TT*GG
    3 POL3=T**4*GA(3,N)
      POLL3=TT**4*GA(3,N)+3.DO*EL1*TT**3*GA(2,N)+3.DO*EL1**2*TT**2
     1*GA(1,N)+EL1**3*TT*GG
      POL2=T**3*GA(2,N)
      POLL2=TT**3*GA(2,N)+2.DO*EL1*TT**2*GA(1,N)+EL1**2*TT*GG
      POL1=T**2*GA(1,N)
      POLL1=TT**2*GA(1,N)+EL1*TT*GG
      A1(N)=2.DO/EL1*(C6*POL6+C5*POL5+C4*POL4+C3*POL3+C2*POL2+C1*POL1
     1-FC*POL1+FD*POL3)
```

```
      A2(N)=2.D0/EL2*(C6*POLL6+C5*POLL5+C4*POLL4+C3*POLL3+C2*POLL2
     1+C1*POLL1-FC*POLL1+FD*POLL3-FE*TT**4*GA(3,N))
   55 CONTINUE
   57 CONTINUE
      DO 56 I=1,NFOR
      IF(KK.EQ.1) A(I)=A1(I)
   56 IF(KK.EQ.2) A(I)=A2(I)
      P=PC-1.D0
      IF(MM.EQ.2) GO TO 11
      IF(ACOUNT.GE.2) GO TO 11
      IF(BCOUNT.GE.2) GO TO 11
      IF(MMM.GE.2) GO TO 11
C
C           COMPUTE INITIAL DEFLECTION
C
      X(1)=0.D0
      Y(1)=0.D0
      MI=L/2+1
      DO 50 II=2,MI
      SUM=0.D0
      X(II)=X(II-1)+2
      DO 60 J=1,NFOR
      EL=L
      EJ=J
      RTEST=DSIN(EJ*PI*X(II)/EL)
      YY=A(J)*RTEST
      SUM=SUM+YY
      Y(II)=SUM
   60 CONTINUE
   50 CONTINUE
      IF(KK.EQ.2) GO TO 14
      DO 65 LL=1,MI
      XC(LL)=X(LL)
   65 YC(LL)=Y(LL)
      GO TO 11
   14 MC=L1/2+2
      MK=L1/2+L/2+1
      DO 66 LL=MC,MK
      ML=L1/2
      XC(LL)=X(LL-ML)+L1
   66 YC(LL)=Y(LL-ML)
      WRITE(6,2002)
 2002 FORMAT('0',10X,'INITIAL DEFLECTION')
      MD=(L1+L2)/2+1
   12 WRITE(6,2001) (XC(M),YC(M),M=1,MD)
 2001 FORMAT('0',10X,F5.1,20X,F20.10)
      DO 70 I=1,MD
      XA(I)=XC(I)
```

```
      YA(I)=YC(I)
   70 CONTINUE
C
C              DRAW INITIAL DEFLECTION CURVE. (MODIFY THIS STATEMENT
C              IF NECESSARY FOR SCALING OR OTHER PURPOSE)
C
      CALL DRAW( MD,XA,YA,1,0,LABEL,ITITLE,50.0,2.0,3,0,2,2,8,6,0,LAST)
C
C             COMPUTE LOAD BY NEWTON-RAPHSON METHOD
C
   11 WRITE(6,1000) EPS
 1000 FORMAT('0',10X,'EPS=',F20.10)
      Q=(U-S)/L
      WRITE(6,6000) S,U
 6000 FORMAT('0',10X,'LEFT END MOMENT=',F10.1,10X,'RIGHT END MOMENT=',
     1F10.1)
   13 COUNT=0.D0
      AE=W*E*EPS
      BE=(PI**2*W*E)/(4.D0*L**2)
  200 COUNT=COUNT+1.D0
      SUMAT=0.D0
      ASUMAT=0.D0
      DO 100 J=1,NFOR
      EJ=J
      EL=L
      H(J)=EJ**2*A(J)*PC+(-1.D0)**J*(2.D0*Q*EL)/(EJ*PI)+((-1.D0)**J-1.D0
     1)*(2.D0*S/(EJ*PI))
      C(J)=EJ**2*PC
      HC(J)=H(J)/(C(J)-P)
      SUMA=EJ**2*(HC(J)**2-A(J)**2)
      SUMAT=SUMAT+SUMA
      ASUMA=(EJ**2*2.D0*H(J)**2)/(C(J)-P)**3
      ASUMAT=ASUMAT+ASUMA
  100 CONTINUE
      FP=P-AE+BE*SUMAT
      FPP=1.D0+BE*ASUMAT
      PP=P-FP/FPP
      ABSVAL=DABS(PP-P)
      IF(ABSVAL.LT.ESP) GO TO 300
      P=PP
      GO TO 200
  300 WRITE(6,5000) COUNT,PP
 5000 FORMAT('0',10X,'COUNT=',F5.1,18X,'LOAD=',F20.10)
C
C             COMPUTE B COEFFICIENTS
C
      DO 20 J=1,NFOR
      EJ=J
```

```
      B(J)=(EJ**2*A(J)*PC+(-1.D0)**J*(2.D0*Q*EL/(EJ*PI))+((-1.D0)**J-1.D
     10)*(2.D0*S/(EJ*PI)))/(EJ**2*PC-PP)
   20 CONTINUE
      IF(MM.EQ.2) GO TO 16
C
C           COMPUTE END SLOPES
C
      ASUM=0.D0
      BSUM=0.D0
      DO 88 JJ=1,NFOR
      BJ=JJ
      BL=L
      RAP=BJ*PI*B(JJ)/BL
      ASUM=ASUM+RAP
      THETAL=ASUM
      BAP=(-1.D0)**JJ*BJ*PI*B(JJ)/BL
      BSUM=BSUM+BAP
      THETAR=BSUM
   88 CONTINUE
      WRITE(6,9002) THETAL,THETAR
 9002 FORMAT('0',10X,'LEFT END SLOPE=',F20.10,5X,'RIGHT END SLOPE=',F20.
     110)
      IF(KK.EQ.1) GO TO 51
      IF(KK.EQ.2) GO TO 52
   51 THETER(KK)=THETAR
      WRITE(6,1011) THETER(KK)
 1011 FORMAT('0',10X,'THETA RIGHT ONE=',F20.10)
      GO TO 22
   52 THETEL(KK)=THETAL
      WRITE(6,1012) THETEL(KK)
 1012 FORMAT('0',10X,'THETA LEFT TWO=',F20.10)
      GO TO 22
   16 CONTINUE
C
C           COMPUTE FINAL DEFLECTION
C
      XN(1)=0.D0
      YN(1)=0.D0
      NI=L/2+1
      DO 500 II=2,NI
      SUM=0.D0
      XN(II)=XN(II-1)+2
      DO 600 J=1,NFOR
      EL=L
      EJ=J
      RTEST=DSIN(EJ*PI*XN(II)/EL)
      YY=B(J)*RTEST
      SUM=SUM+YY
```

```
      YN(II)=SUM
  600 CONTINUE
  500 CONTINUE
      IF(KK.EQ.2) GO TO 140
      DO 650 LL=1,NI
      XM(LL)=XN(LL)
  650 YM(LL)=YN(LL)
      GO TO 22
  140 NB=L1/2+2
      NK=L1/2+L/2+1
      DO 660 LL=NB,NK
      NL=L1/2
      XM(LL)=XN(LL-NL)+L1
  660 YM(LL)=YN(LL-NL)
      WRITE(6,2003)
 2003 FORMAT('0',10X,'FINAL DEFLECTION')
      NN=(L1+L2)/2+1
      WRITE(6,2004) (XM(M),YM(M),M=1,NN)
 2004 FORMAT('0',10X,F5.1,20X,F20.10)
      DO 700 I=1,NN
      XA(I)=XM(I)
      YA(I)=YM(I)
  700 CONTINUE
C
C             DRAW FINAL DEFLECTION CURVE. (MODIFY THIS STATEMENT
C             IF NECESSARY FOR SCALING OR OTHER PURPOSE)
C
      CALL DRAW(NN,XA,YA,NG,0,LABEL,ITITLE,50.0,2.0,3,0,2,2,8,6,0,LAST)
      GO TO 90
   22 CONTINUE
C
C             COMPUTE ERROR IN SLOPE AT JUNCTION
C
      ERR=DABS(THETER(1)-THETEL(2))
      WRITE(6,1003) ERR
 1003 FORMAT('0',10X,'ERROR=',F20.10)
      WRITE(6,4444)
 4444 FORMAT('0',20X,'*************************************')
      IF(DABS(ERR).GT.DABS(ERRO)) GO TO 96
      XMOM=XMOM-DELT
      ERRO=ERR
      GO TO 80
   96 DELTF=DELTF/1.D1
      XMOM=XMOM+DELT
      BCOUNT=BCOUNT+1.D0
      IF(BCOUNT.EQ.NDEC) GO TO 81
      GO TO 80
   81 WRITE(6,7771) DELT
```

```
7771 FORMAT('0',10X,'DELTA=',F10.1)
     WRITE(6,7772) XMOM,ERR
7772 FORMAT('0',10X,'CENTER MOMENT=',F20.10,10X,'SLOPE ERROR=',F20.10)
  90 CONTINUE
     SS=SS-100.D3
  99 CONTINUE
     STOP
     END

     SUBROUTINE SINCOE(M,N,R)
C    THIS SUBROUTINE EVALUATES INTEGRAL ZERO TO PI OF X**M*DSIN(EN*X)
C    WHERE EN=N. RESULT IS R.
     IMPLICIT REAL*8 (A-H,O-Z)
     PI=4.D0*DATAN(1.D0)
     I=M/2
     K=M-2*I
     O=1.D0
     EN=N
     EM=M
     S=(EN*PI)**M
     MF=1
     DO 3 KK=1,M
   3 MF=MF*KK
     EMF=MF
     NF=O
     DO 4 J=1,I
     NF=NF*(M+1-2*J)*(M+2-2*J)
     F=NF
   4 S=S+(-O)**J*F*(EN*PI)**(M-2*J)
     S=S*(-O)**(N+1)
     IF(K.EQ.0) S=S+(-O)**I*EMF
     R=S*EN**(-(M+1))
     RETURN
     END
```

BIBLIOGRAPHY

1. Brock, John E., "Elastic Buckling of Heated, Straight-Line Piping Configurations," Oak Ridge National Laboratory Report ORNL-TM-3607, March 1972.

2. Fung, Y. C., and Kaplan, A., "Buckling of Low Arches or Curved Beams of Small Curvature," NACA Technical Note 2840, 1952.

3. Gjelsvik, A., and Bodner, S. R., "Energy Criterion and Snap Buckling of Arches," Journal of Engineering Mechanics Division, American Society of Civil Engineers, Vol. 92, p. 1-19, 1966.

4. Grobner, W., and Hofreiter, N., Integraltafel, Vol. 11, Definite Integrals, Springer-Verlag, Vienna and New York, 1966.

5. Hoff, N. J., and Bruce, Victor G., "Dynamic Analysis of the Buckling of Laterally Loaded Flat Arches," Journal of Mathematics and Physics, Vol. XXXII, No. 4, p. 276-288, Jan. 1954.

6. Hsu, C. S., "Stability of Shallow Arches Against Snap-Through Under Timewise Step Loads," Journal of Applied Mechanics, p. 31-39, March 1968.

7. Huddleston, John V., "Effect of Axial Strain on Buckling and Postbuckling Behavior of Elastic Columns," Proceedings of the Fourth Southeastern Conference on Theoretical and Applied Mechanics, New Orleans, La., p. 263-273, Feb. 1968.

8. Timoshenko, S., Theory of Elastic Stability, p. 204-238, McGraw-Hill Book Co., Inc., 1936.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Professor John E. Brock, Code 59Bc Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93940	1
4.	Department of Mechanical Engineering, Code 59 Naval Postgraduate School Monterey, California 93940	1
5.	LT Robert B. Ploeger, USN P. O. Box 476 Darien, Georgia 31305	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (*Corporate author*) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
Analysis of Initially Bent Two-Span Columns

4. DESCRIPTIVE NOTES (*Type of report and, inclusive dates*)
Master's Thesis, June 1972

5. AUTHOR(S) (*First name, middle initial, last name*)
Robert B. Ploeger

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1972	57	8

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (*Any other numbers that may be assigned this report*)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

A study of single-span initially bent columns is made and extended to include two-span columns. The axial force in these columns results from heating (or an equivalent) and from bowing. A snap-through phenomenon is considered in the analysis of the single-span and observations are made concerning snap-through in the two-span column. A digital computer program is developed to carry out the analysis and a description of the program enables its use by anyone familiar with the FORTRAN language.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
olumn						
wo-span column						
nap-through						
nitially bent column						
eated column						

Thesis 135160
P6156 Ploeger
c.1 Analysis of initially
bent two-span columns.